The axial coupling at sub-percent precision from lattice QCD

Chris Monahan
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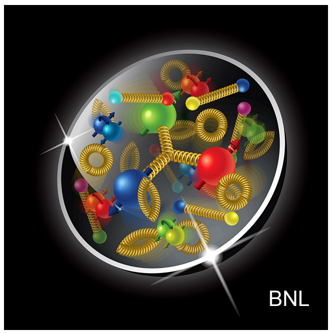
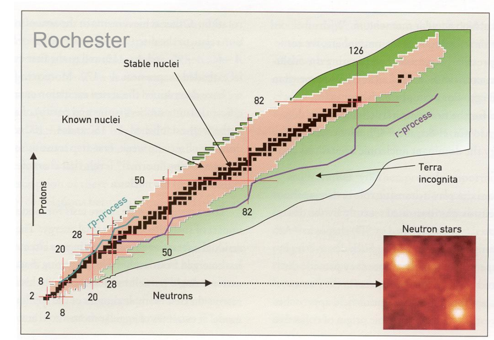
California Lattice Collaboration

Anchoring nuclear physics in QCD

First principles' calculations of nucleon properties

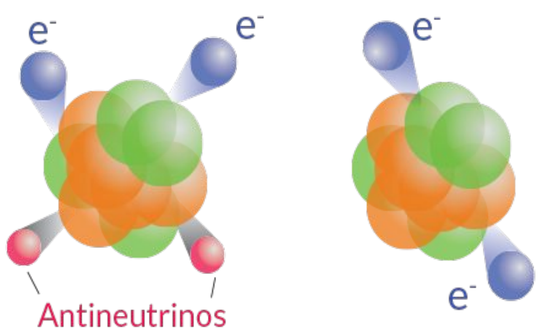
- program to anchor low energy nuclear physics in QCD
- requires lattice QCD, effective theories and many body nuclear theory

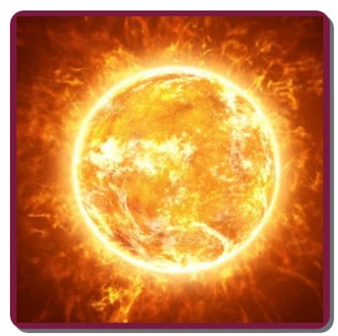


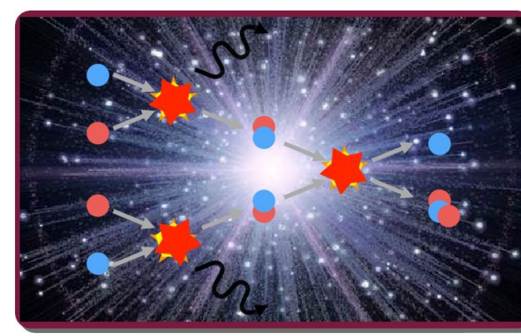


Start with the simplest properties, e.g. axial coupling of nucleon

- Fundamental parameter of nuclear physics







- Benchmark for ab initio calculations

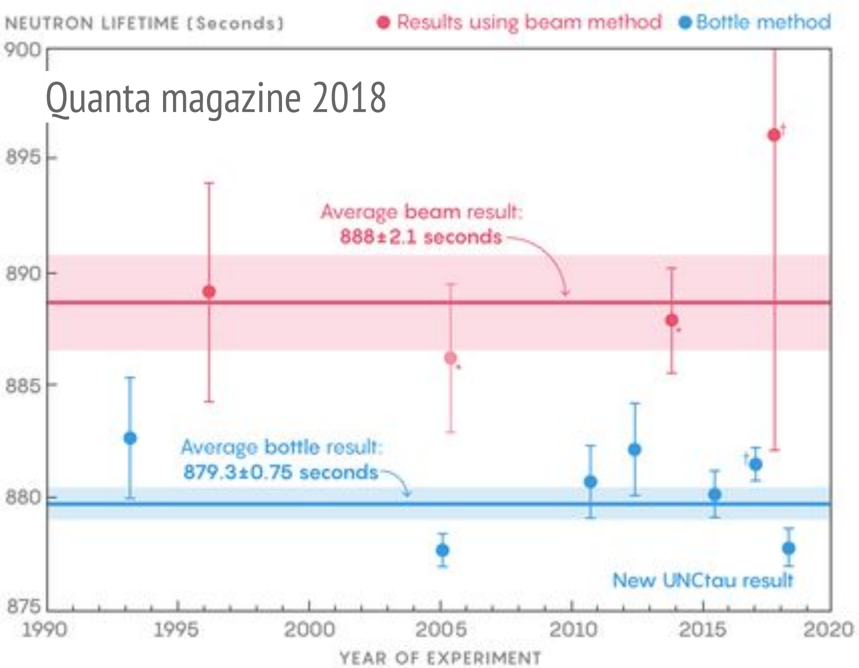
e.g. Edwards et al., PRL 96 (2006) 062001

Neutron lifetime puzzle

Long-standing tension (until recently?) in measurements of the neutron lifetime

$$\tau_n^{\text{beam}} = 888.0(2.0)s$$

$$\tau_n^{\text{bottle}} = 879.4(0.6)s$$



Neutron lifetime directly tied to axial coupling

$$|V_{ud}|^2 \tau_n (1 + 3g_A^2) = 4906(1.7)\,\mathrm{s}$$

Czarnecki et al., 1907.06737

Czarnecki et al., PRL 120 (2018) 202002

Matching larger uncertainty from beam experiments requires < 0.2% precision

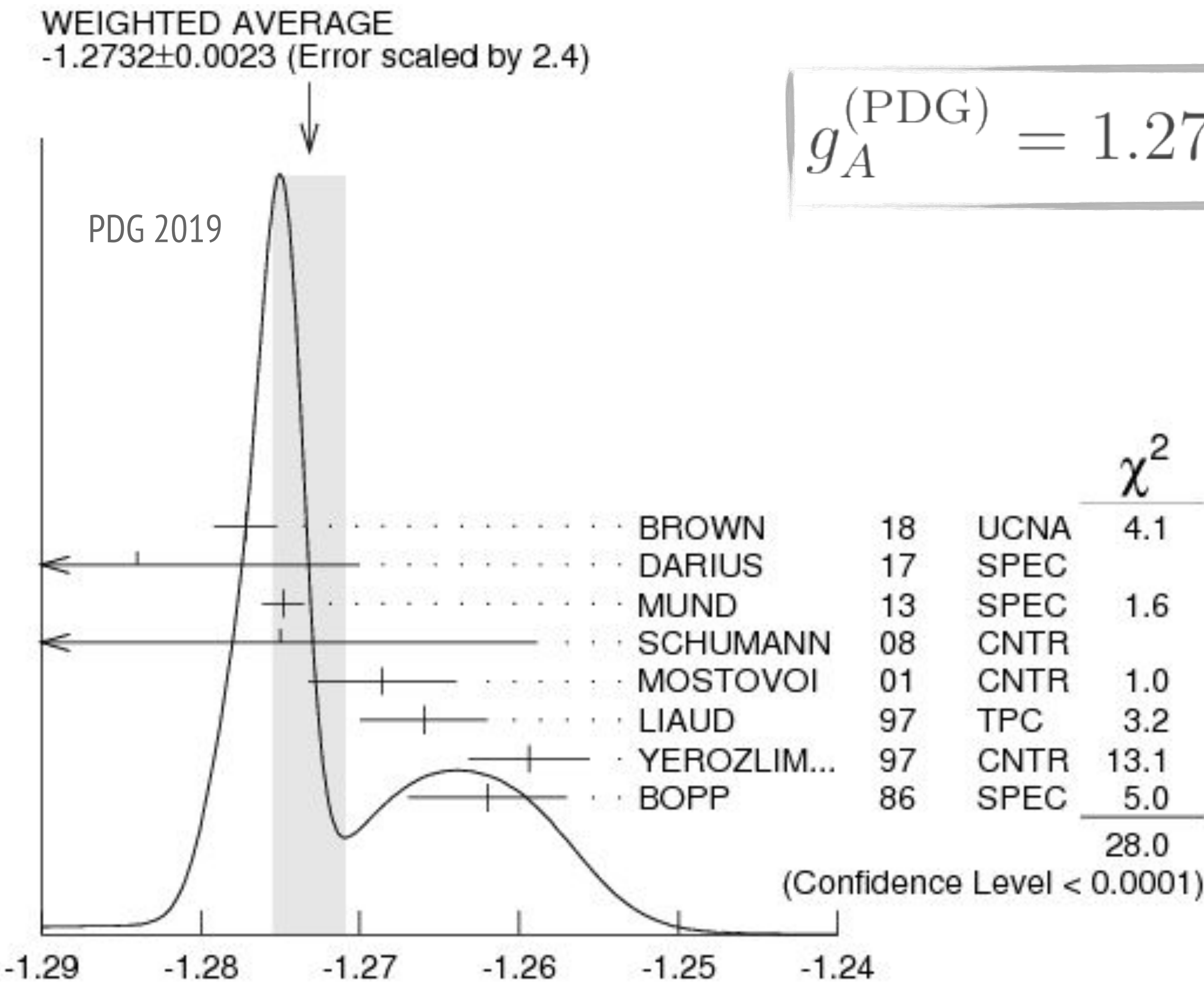
Story has become more subtle

- This year, PDG dropped beam measurements completely
- Radiative corrections still under investigation
- Matching the (more precise) bottle measurements requires ~ 0.05% precision
- Matching most precise axial coupling measurements requires ~ 0.02% precision

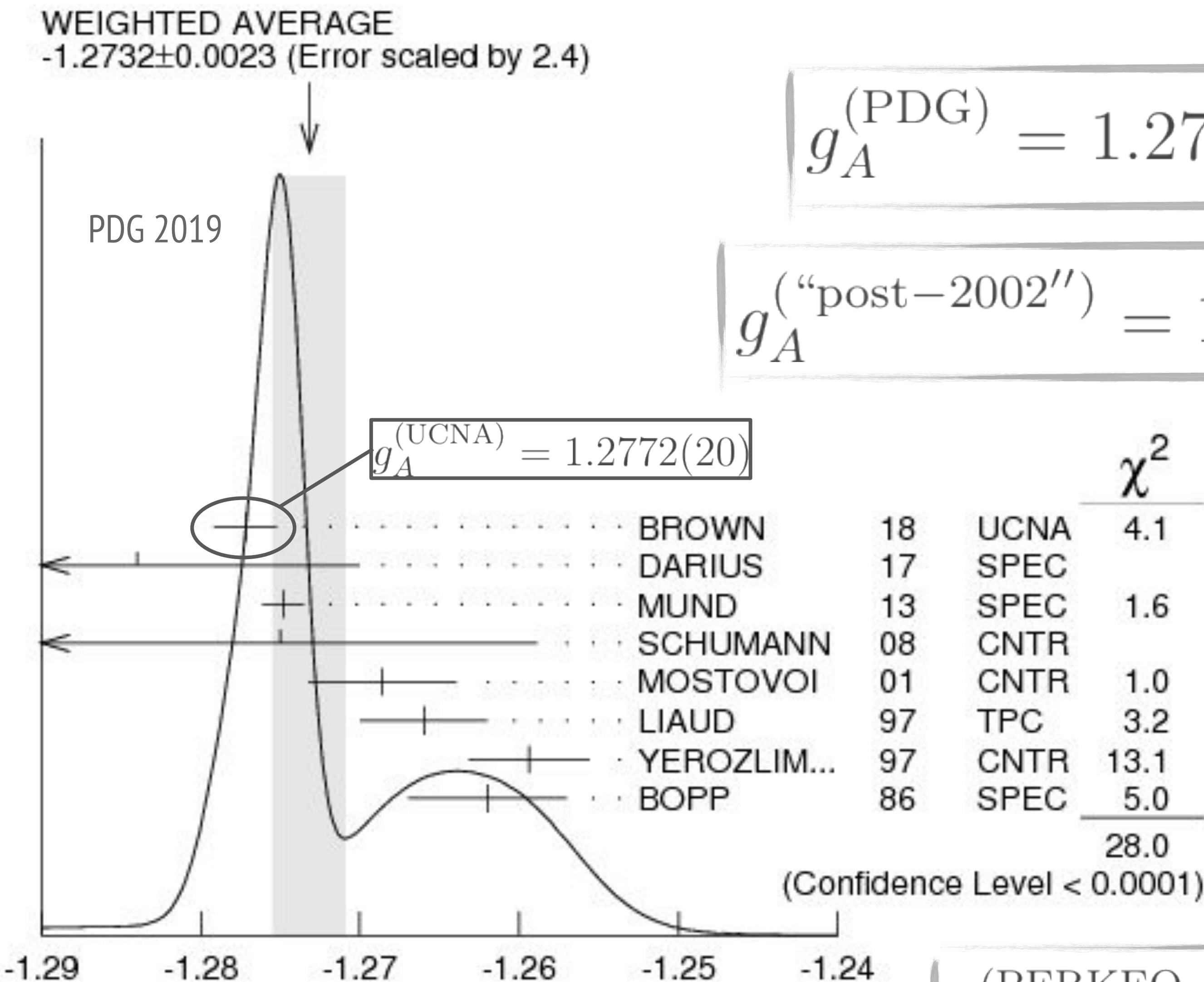
See also Seng et al., PRD 100 (2019) 013001

Axial coupling from experiment

$$g_A^{(\mathrm{PDG})} = 1.2732(23)$$



Axial coupling from experiment



$$g_A^{(\mathrm{PDG})} = 1.2732(23)$$

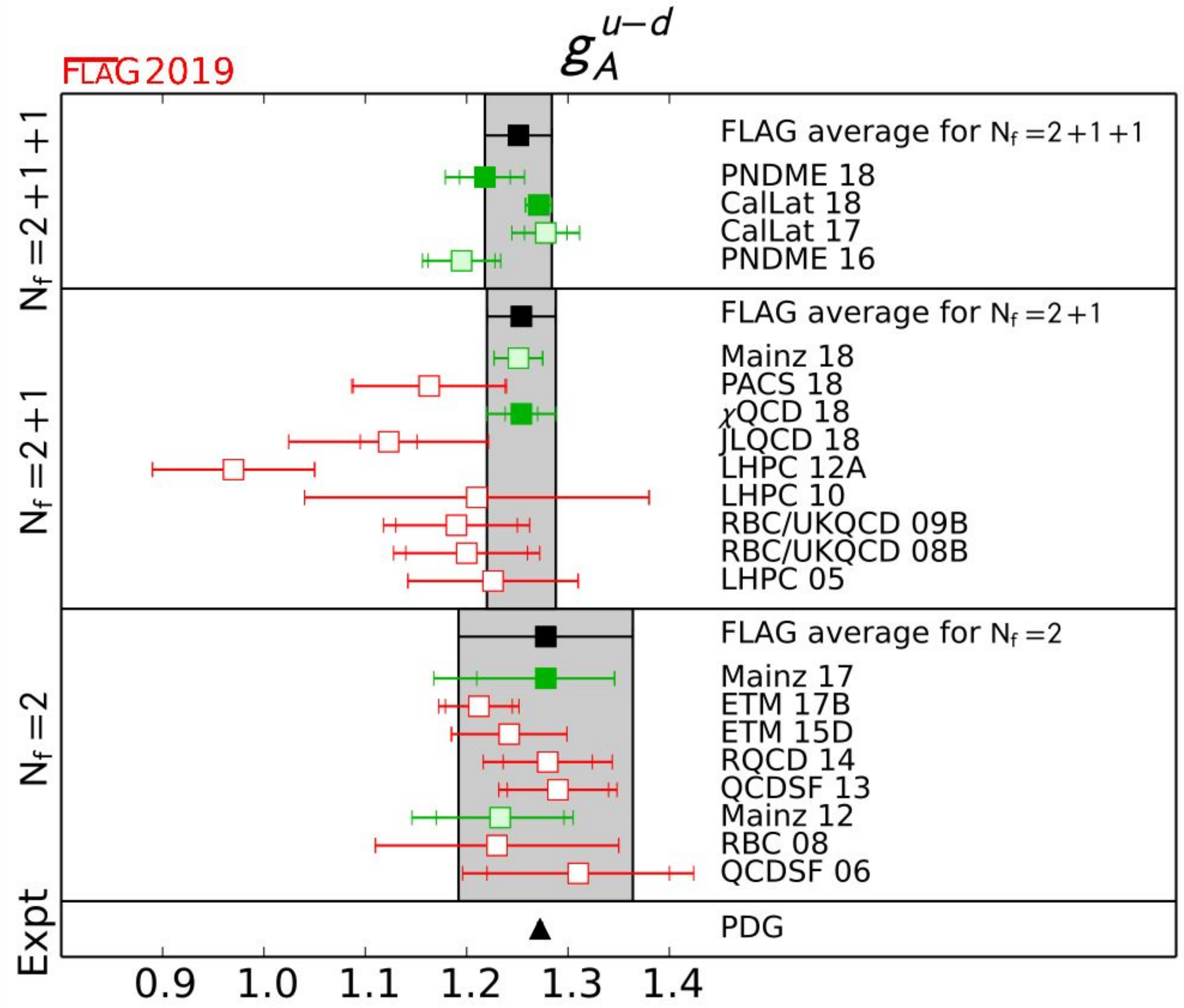
$$g_A^{(\text{“post}-2002\text{''})} = 1.2762(5)$$

Czarnecki et al., 1907.06737

$$g_A^{(\mathrm{PERKEO-III})} = 1.2764(6)$$

Markisch et al., PRL 122 (2019) 242501

Axial coupling on the lattice

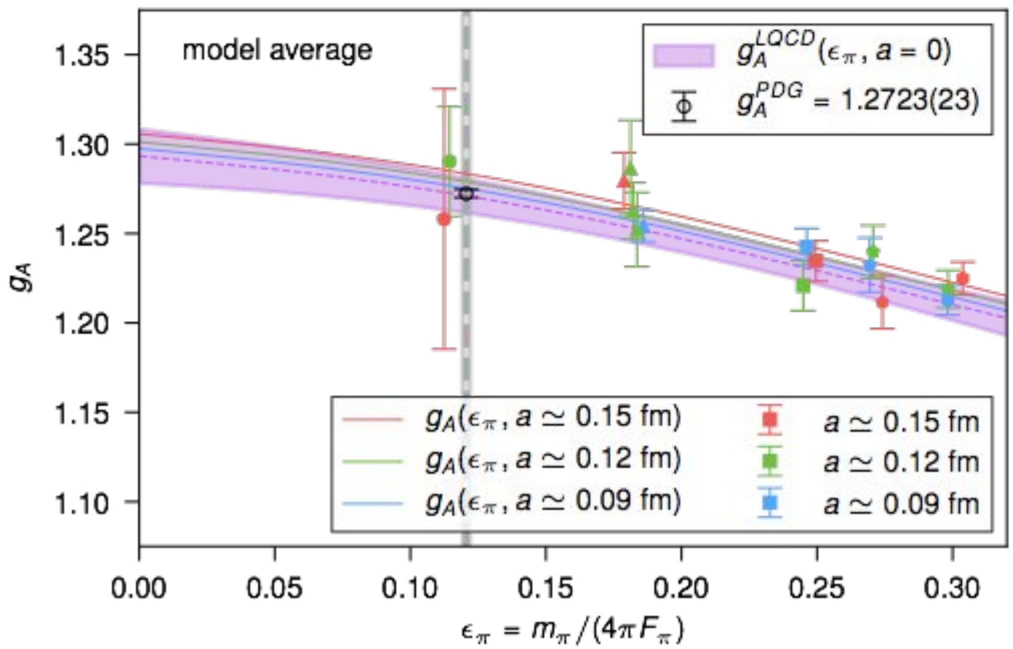


FLAG 2019, 1902.08191

Axial coupling from CalLat and friends

First calculation at one percent precision

$$g_A^{\rm QCD} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$



statistical	0.81%
chiral extrapolation	0.31%
$a \to 0$	0.12%
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model selection	0.43%
total	0.99%

Chang et al., Nature 558 (2018) 91

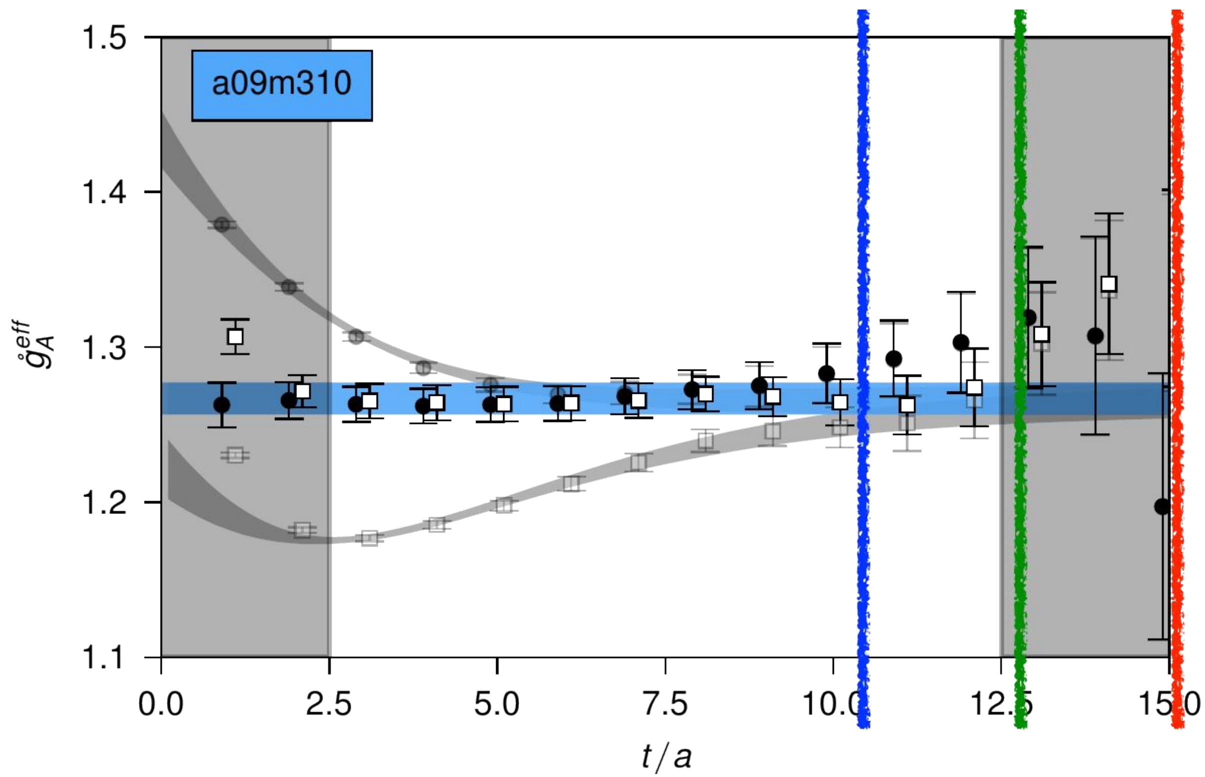
Axial coupling from CalLat and friends

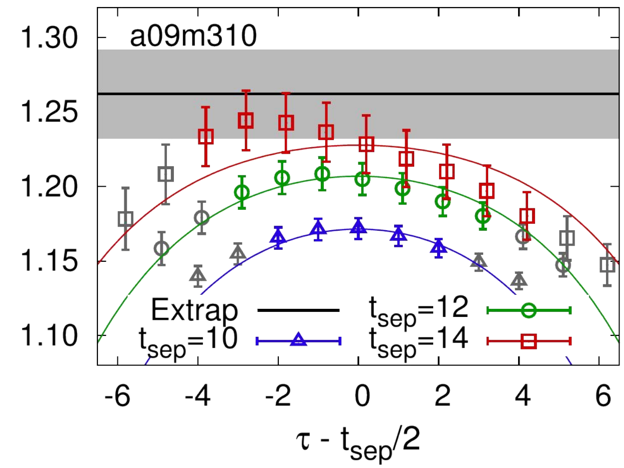
High precision enabled by:

Bouchard et al., PRD 96 (2017) 014504

1. Feynman-Hellmann inspired method that exploits exponentially more precise data at early Euclidean times, with demonstrable control of excited state contributions

PNDME, PRD 94 (2016) 054508





$$\left.\frac{\partial m^{\text{eff.}}(t,\tau)}{\partial \lambda}\right|_{\lambda=0} = -\frac{1}{\tau}\left[\frac{\partial_\lambda C(t+\tau)}{C(t+\tau)} - \frac{\partial_\lambda C(t)}{C(t)}\right]$$

History of similar ideas

Savage et al., PRL 119 (2017) 062002

Chambers et al., PRD 90 (2014) 014510

Chambers et al., PRD 92 (2015) 114517

Bulava et al., JHEP 1201 (2012)

De Diviitis et al., PLB 718 (2012)

Güsken et al., PLB 227 (1989)

Maiani et al., NPB 293 (1987)

Axial coupling from CalLat and friends

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2. Mixed lattice action with: improved stochastic behaviour, very mild continuum extrapolation and highly suppressed chiral symmetry breaking.
   Berkowitz et al., PRD 96 (2017) 054513

Axial coupling from CalLat and friends

High precision enabled by:

3. Access to ensembles (MILC) that allowed control over lattice systematics.

HISQ gauge configuration parameters							valence parameters							
abbr.	$N_{\rm cfg}$	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5} L$	$N_{\rm src}$	L_5/a	aM_5	b_5	c_5	$am_l^{\rm val.}$	$\sigma_{\rm smr}$	$N_{\rm smr}$
a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8	24	12	1.3	1.5	0.5	0.01580	4.2	60
a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0	12	16	1.3	1.75	0.75	0.00712	4.5	60
a15m130	1000	$32^3 \times 48$	0.15	0.036	130	3.2	5	24	1.3	2.25	1.25	0.00216	4.5	60
a12m310	1053	$24^3 \times 64$	0.12	0.2	310	4.5	8	8	1.2	1.25	0.25	0.01260	3.0	30
a12m220S	1000	$24^3 \times 64$	0.12	0.1	220	3.2	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m220	1000	$32^3 \times 64$	0.12	0.1	220	4.3	4	12	1.2	1.5	0.5	0.00600	6.0	90
a12m220L	1000	$40^3 \times 64$	0.12	0.1	220	5.4	4	12	1.2	1.5	0.5	0.00600	6.0	90
a09m310	784	$32^3 \times 96$	0.09	0.2	310	4.5	8	6	1.1	1.25	0.25	0.00951	7.5	167

Berkowitz et al., 1704.01114

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	HISQ gauge configuration parameters							valence parameters							
	abbr.	N_{cfg}	volume	$\sim a$ [fm]	m_l/m_s	$\sim m_{\pi_5}$ [MeV]	$\sim m_{\pi_5} L$	N_{src}	L_5/a	aM_5	b_5	c_5	$am_l^{\mathrm{val.}}$	σ_{smr}	N_{smr}
*	a15m400	1000	$16^3 \times 48$	0.15	0.334	400	4.8	8	12	1.3	1.5	0.5	0.0278	3.0	30
*	a15m350	1000	$16^3 \times 48$	0.15	0.255	350	4.2	16	12	1.3	1.5	0.5	0.0206	3.0	30
	a15m310	1960	$16^3 \times 48$	0.15	0.2	310	3.8	24	12	1.3	1.5	0.5	0.01580	4.2	60
	a15m220	1000	$24^3 \times 48$	0.15	0.1	220	4.0	12	16	1.3	1.75	0.75	0.00712	4.5	60
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*	a12m400	1000	$24^3 \times 64$	0.12	0.334	400	5.8	8	8	1.2	1.25	0.25	0.02190	3.0	30
*	a12m350	1000	$24^3 \times 64$	0.12	0.255	350	5.1	8	8	1.2	1.25	0.25	0.01660	3.0	30
	a12m310	1053	$24^3 \times 64$	0.12	0.2	310	4.5	8	8	1.2	1.25	0.25	0.01260	3.0	30
	a12m220S	1000	$24^3 \times 64$	0.12	0.1	220	3.2	4	12	1.2	1.5	0.5	0.00600	6.0	90
	a12m220	1000	$32^3 \times 64$	0.12	0.1	220	4.3	4	12	1.2	1.5	0.5	0.00600	6.0	90
	a12m220L	1000	$40^3 \times 64$	0.12	0.1	220	5.4	4	12	1.2	1.5	0.5	0.00600	6.0	90
*	a12m130	1000	$48^3 \times 64$	0.12	0.036	130	3.9	3	20	1.2	2.0	1.0	0.00195	7.0	150
*	a09m400	1201	$32^3 \times 64$	0.09	0.335	400	5.8	8	6	1.1	1.25	0.25	0.0160	3.5	45
*	a09m350	1201	$32^3 \times 64$	0.09	0.255	350	5.1	8	6	1.1	1.25	0.25	0.0121	3.5	45
	a09m310	784	$32^3 \times 96$	0.09	0.2	310	4.5	8	6	1.1	1.25	0.25	0.00951	7.5	167
*	a09m220	1001	$48^3 \times 96$	0.09	0.1	220	4.7	6	8	1.1	1.25	0.25	0.00449	8.0	150

* New calculation

Additional HISQ ensembles generated at LLNL

Chang et al., Nature 558 (2018) 91

Axial coupling from CalLat and friends

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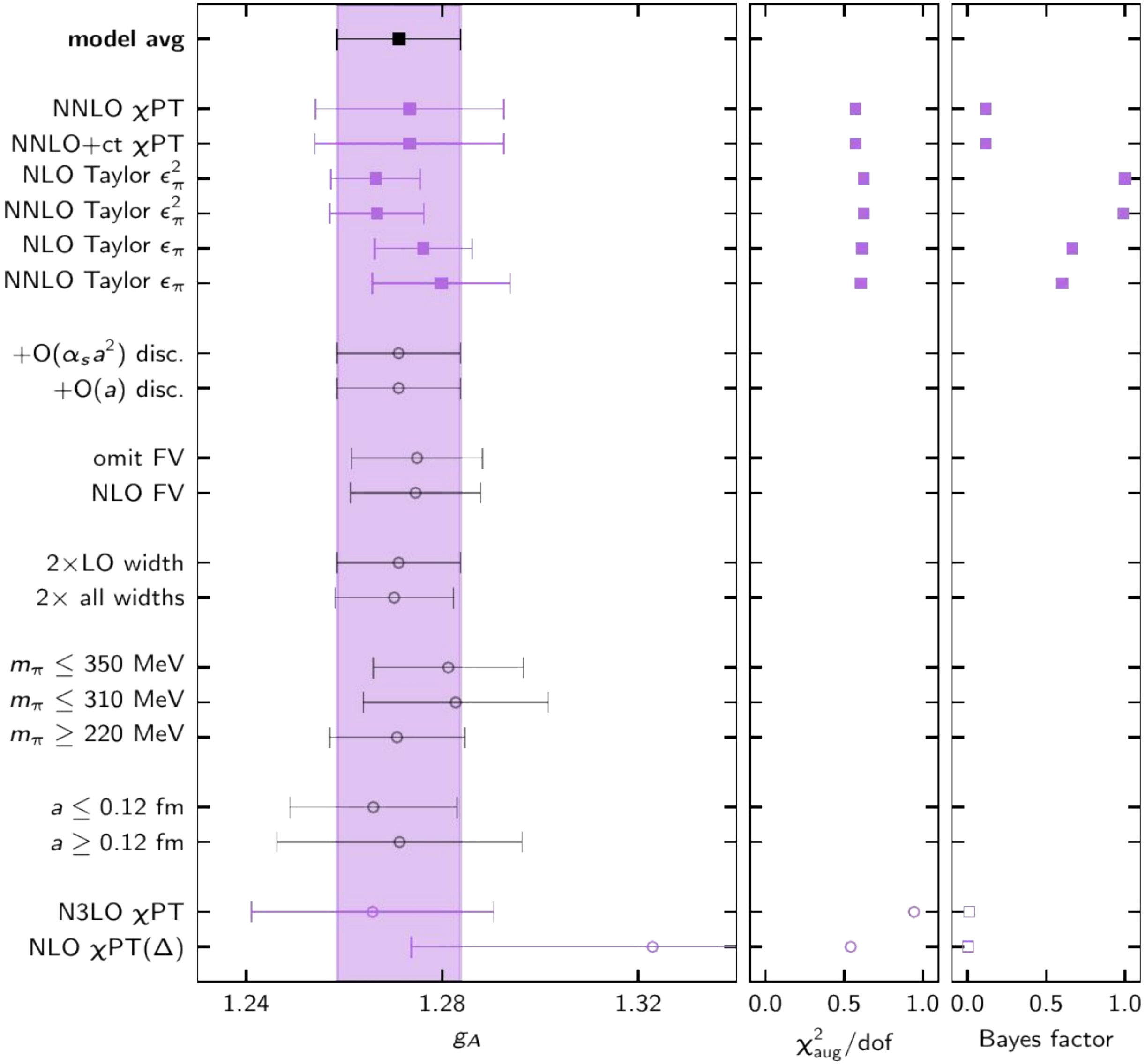
2. Mixed lattice action with: improved stochastic behaviour, very mild continuum extrapolation and highly suppressed chiral symmetry breaking.
   Berkowitz et al., PRD 96 (2017) 054513
3. Access to ensembles (MILC) that allowed control over lattice systematics.
4. Very fast GPU code linking USQCD chroma software suite through the highly optimised QUDA library.
   Joo and Edwards., NPB(PS) 140 (2005) 832
   Clark et al., CPC 181 (2010) 1517
5. Access to leadership class computing.

Axial coupling from CalLat and friends

Worked hard at ensuring stability in fits



final result

6 fits included in the model average and their relative weights

discretisation corrections

finite volume corrections

prior width sensitivity

pion mass cut sensitivity

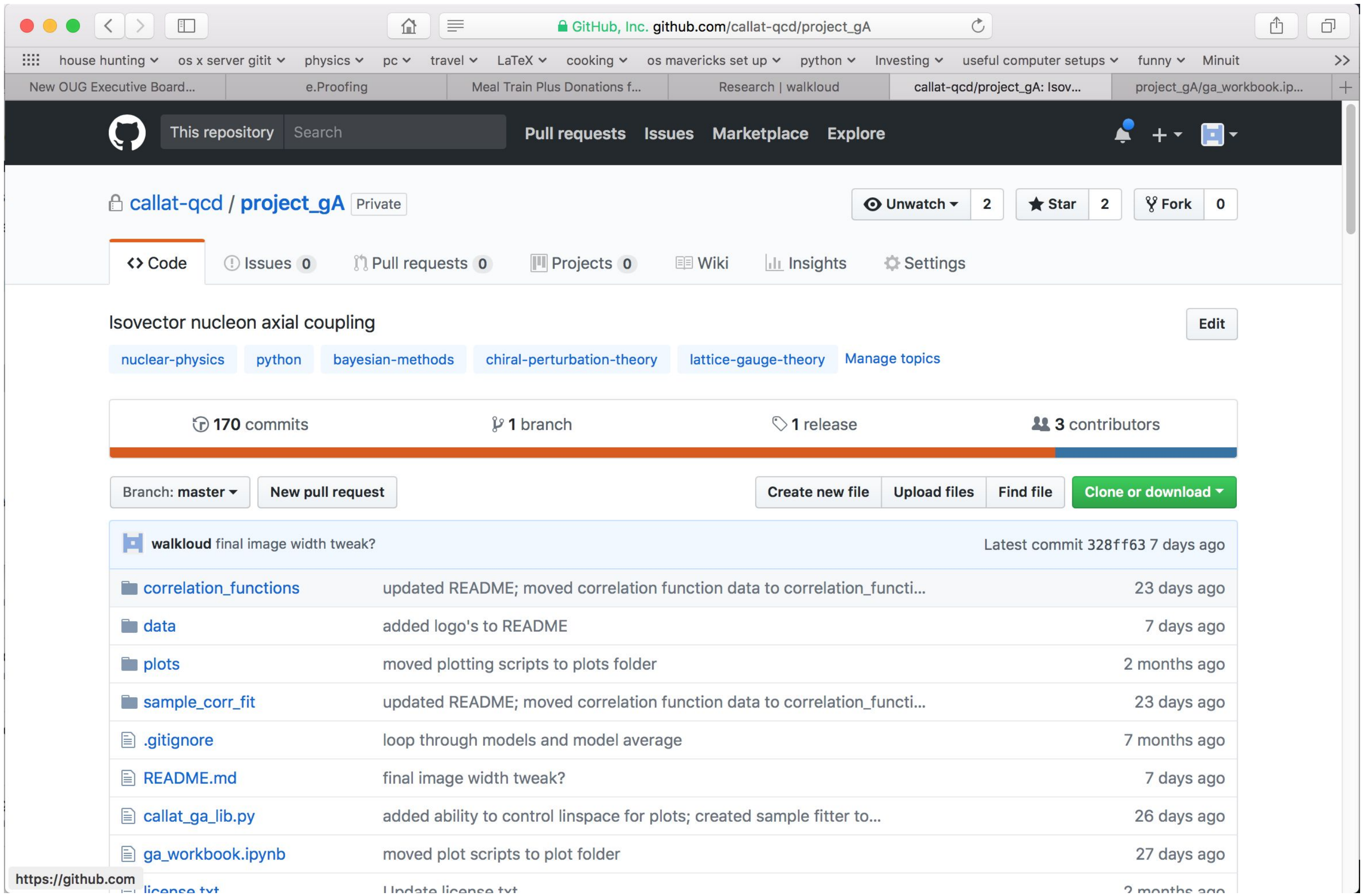
lattice spacing cut sensitivity

additional XPT analysis

Axial coupling from CalLat and friends

All analysis code and data are available online

github.com/callat-qcd/project_gA



We encourage you to play with the data yourself!

Improving the precision

First calculation at one percent precision

$$g_A^{\mathrm{QCD}} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$

Chang et al., Nature 558 (2018) 91

Uncertainty dominated by statistical precision

More precise data at physical pion mass will improve dominant uncertainties

- Statistical (s)
- Chiral extrapolation (χ)
- Model selection (M)

statistical	0.81%
chiral extrapolation	0.31%
$a \to 0$	0.12%
$L \to \infty$	0.15%
isospin	0.03%
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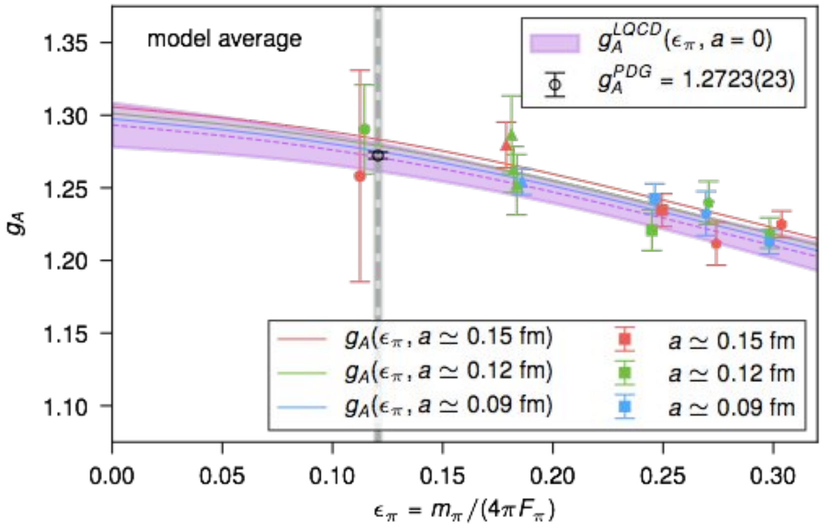
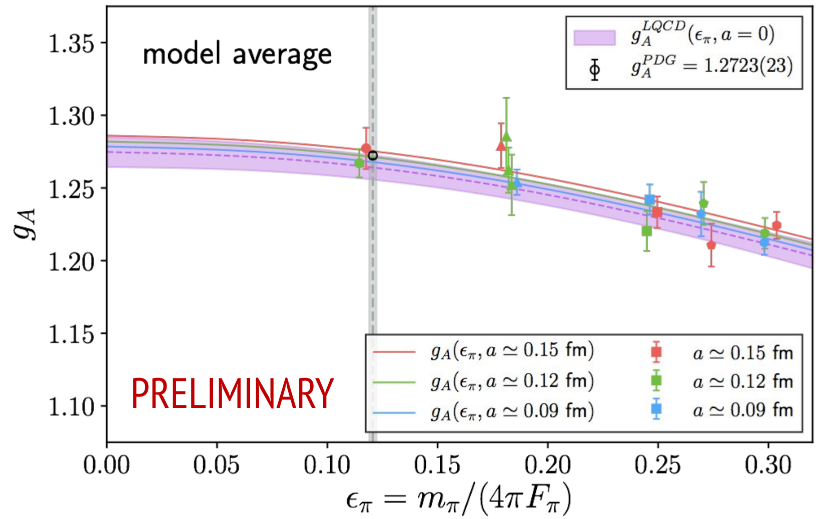
total	0.99%

Improving the precision

Improvements (mostly on Sierra [Early Science])

- 32 sources on a12m130 lattice (up from 3)
- Generated new a15m135XL lattice (48^3x64 vs 32^3x48)





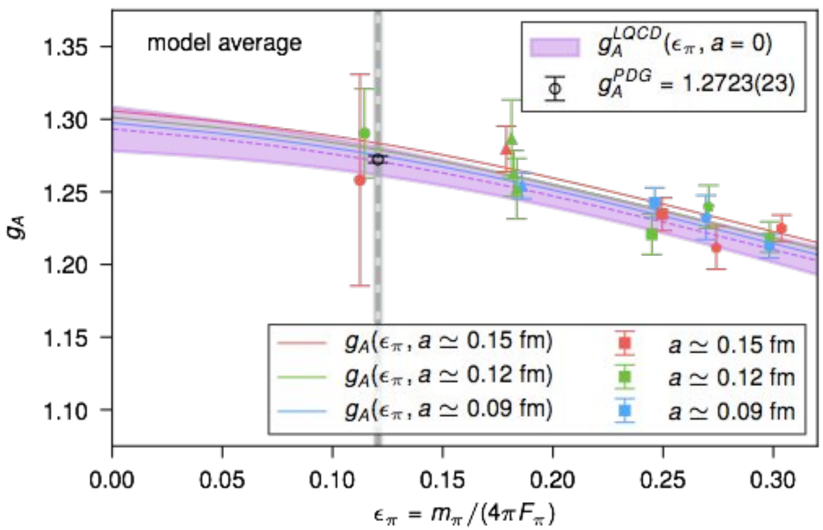
$$g_A = 1.2711(125) \longrightarrow g_A^{(\mathrm{PRELIM.})} = 1.2641(93)$$

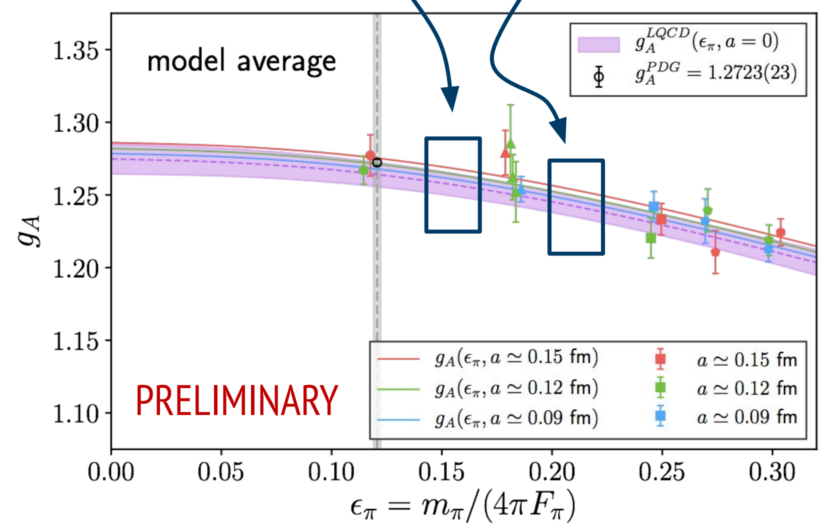
Anticipate ~0.6% precision by the end of the year using current strategy

Improving the precision

Improvements (mostly on Sierra [Early Science])

- 32 sources on a12m130 lattice (up from 3)
- Generated new a15m135XL lattice (48^3x64 vs 32^3x48)
- We are also generating new ensembles at **180 MeV** and **260 MeV**





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Beyond 0.5%?

Moving beyond 0.5-0.6% precision will require

- Adding intermediate pion masses
- Fourth lattice spacing (~ 0.06 fm)
- Finite volume studies at other masses
- Directly incorporating isospin breaking

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Isospin and QED corrections...

- 0.03% estimate comes from ambiguity in extrapolation

$$\epsilon_{\pi^-} = \frac{m_{\pi^-}}{4\pi F_{\pi^-}} \qquad \epsilon_{\pi^0} = \frac{m_{\pi^0}}{4\pi F_{\pi^0}}$$

- corrections from isospin breaking estimated as

$$\mathcal{O}\left(\frac{(m_d - m_u)^2}{(m_d + m_u)^2}\epsilon_\pi^4\right) \sim 0.002\% \qquad \mathcal{O}\left(\alpha_{EM}\frac{m_d - m_u}{m_d + m_u}\epsilon_\pi^2\right) \sim 0.004\%$$

- neglected EW corrections in experimental result

Czarnecki et al., 1907.06737

Czarnecki et al., PRL 120 (2018) 202002

Axial form factor

DUNE - future neutrino oscillation experiment

- one goal is determination of the CP-violating phase in the (PMNS) matrix
- sufficient CP-violation could explain matter-antimatter asymmetry

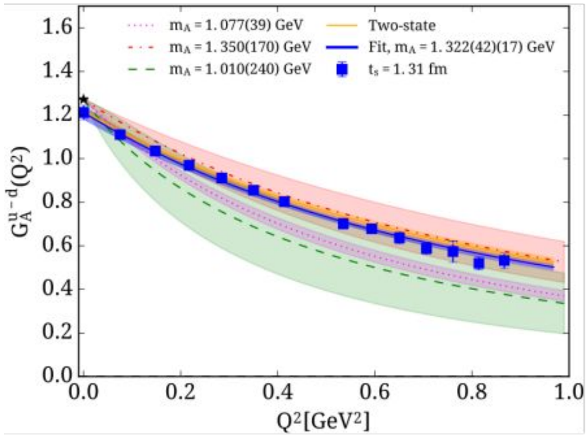
T2K and NOVA are also conducting oscillation experiments

"A determination of the nucleon axial form factor at the 5% level would be very helpful, possibly allowing for the isolation of nuclear effects"
[private communications with T2K members, Y. Hayato and K. Mcfarland]

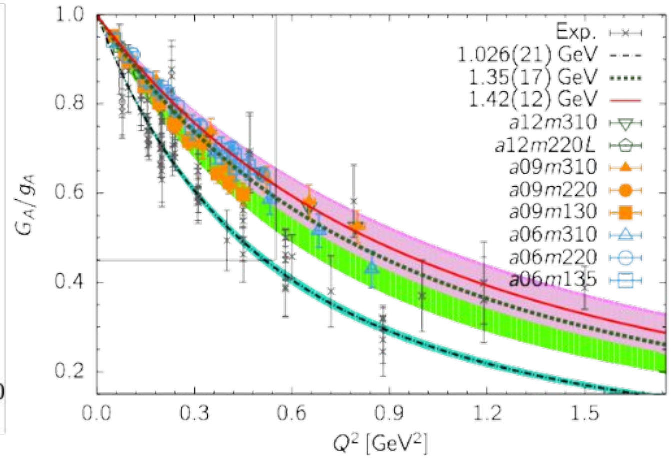
Ultimate aim is neutrino-nucleus cross sections

Experimental data on axial form factor is sufficiently limited that a simple dipole-form factor is usually assumed.

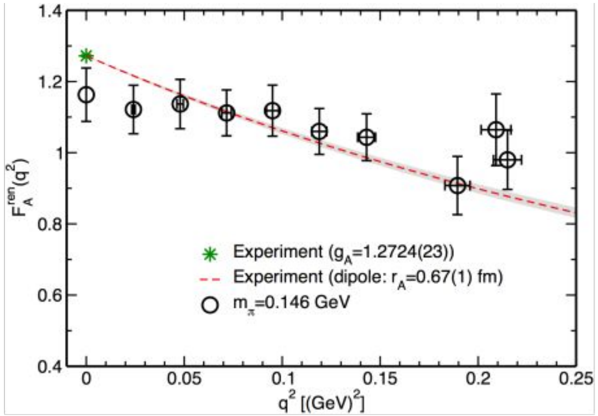
Axial form factor on the lattice



Alexandrou et al., PRD 96 (2017) 054507



Gupta et al., PRD 96 (2017) 114503



Ishikawa et al., PRD 98 (2018) 074510

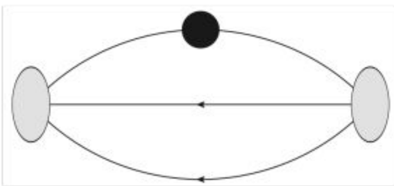
Tension (~30%) between slope determined from lattice QCD and experiment

Unclear where this discrepancy comes from.

Can we apply lessons learned from axial coupling to the form factor?

Challenges

Central to our approach was the “Feynman-Hellmann propagator”



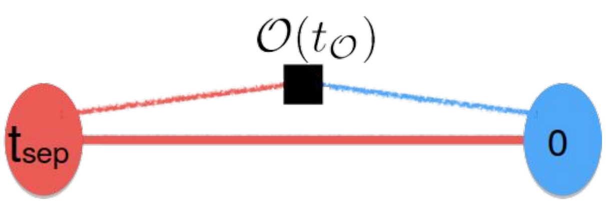
$$= S_{FH}(y,x) = \sum_z S(y,z)\Gamma(z)S(z,x)$$

For each choice of current and momentum, a new FH propagator is required

Tried variants of stochastic methods to relax this constraint

Gambhir et al., PoS(LATTICE2018) 126

Resorted to the standard fixed source-sink separation method



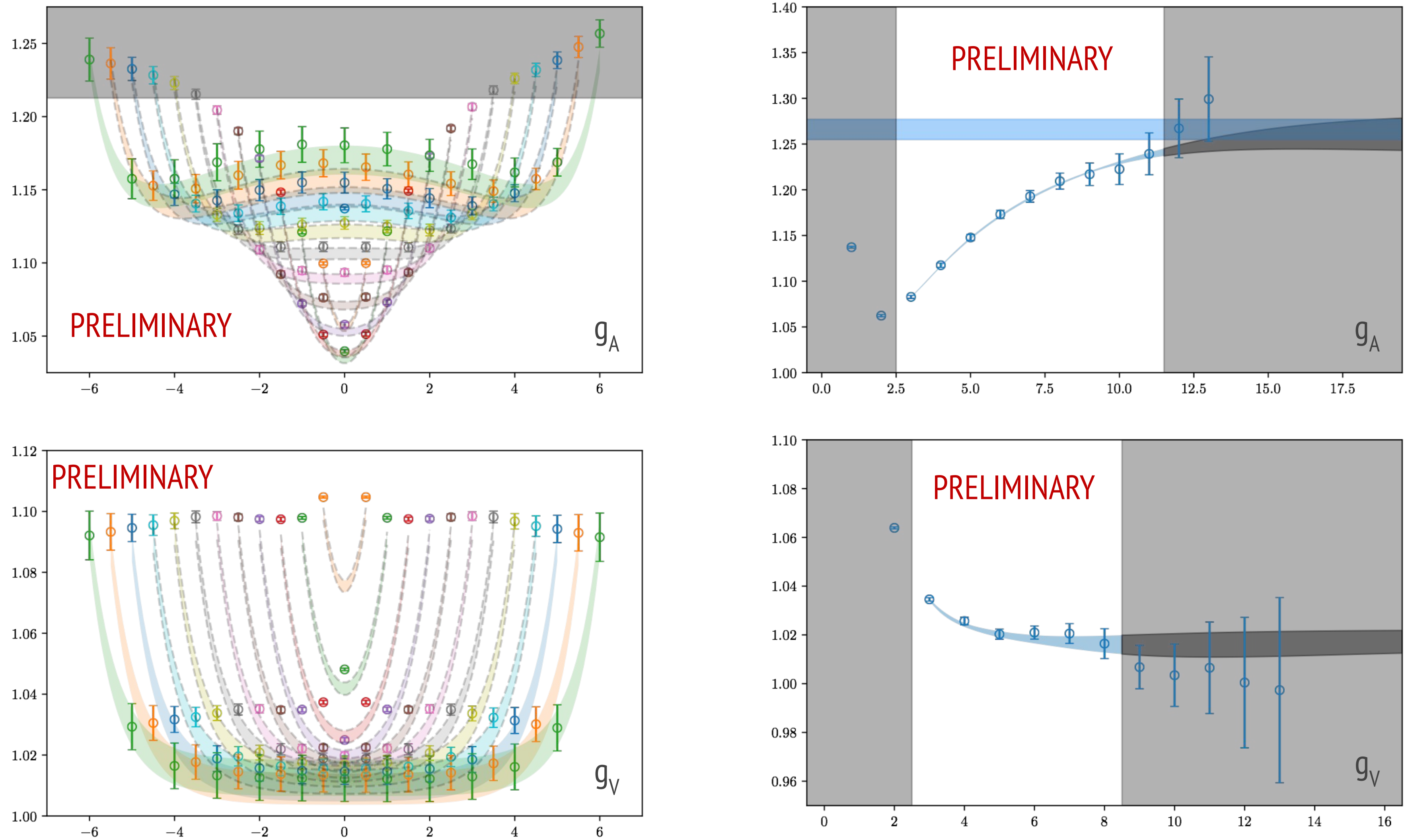
[repeat for multiple values of t_{sep}]

Lesson (for us) from our axial coupling calculation: use many values of t_{sep}

See also S. Meinel, Chiral Dynamics 2012 and Hasan et al., PRD 99 (2019) 114505

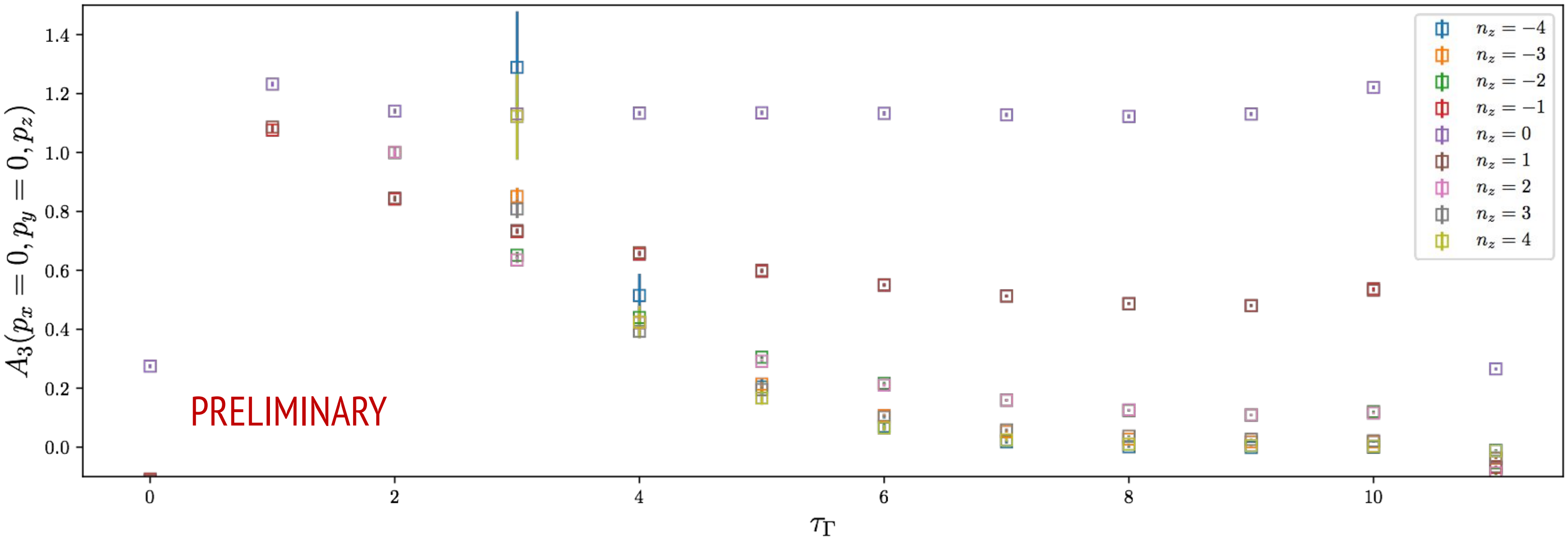
First preliminary results

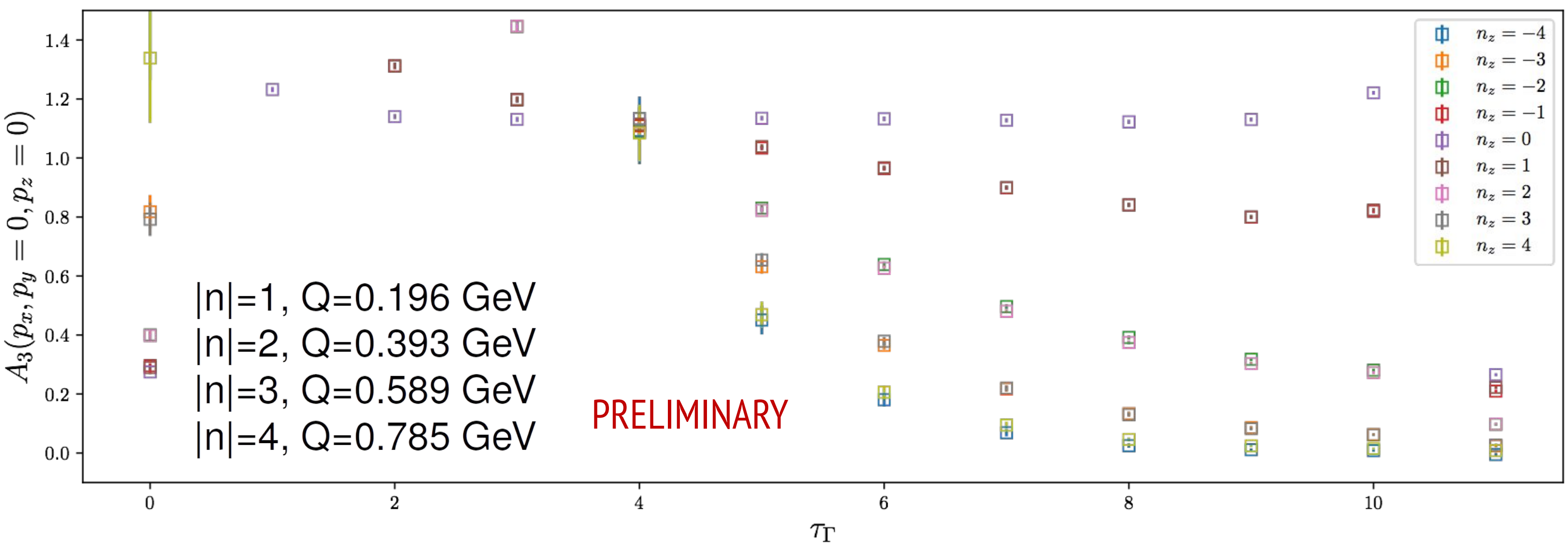
a09m310: t_{sep} = {3,...,14}



First preliminary results

a09m310: t_{sep} = 11, nonzero momentum





Summary

High precision enabled by:

1. Feynman-Hellmann inspired method
2. Mixed lattice action
3. Access to MILC ensembles
4. Very fast GPU code
5. Access to leadership class computing

Uncertainty dominated by statistical uncertainty

- focussed on physical mass ensembles
- on course for ~0.6% uncertainty by the end of the year

Focus now on axial form factor

- employ traditional three-point method with wide range of t_{sep}

California Lattice Collaboration

and friends

Evan Berkowitz
Chris Bouchard
David Brantley
Kate Clark
Henry-Monge-Camacho
Chia Cheng (Jason) Chang
Nicolas Garron
Balint Joo
Thorsten Kurth
Amy Nicholson
Kostas Orginos
Enrico Rinaldi
Andrew Walker-Loud
Pavlos Vranas

Thank you

Chris Monahan
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California Lattice Collaboration

Axial coupling from CalLat and friends

Worked hard at ensuring stability in fits

Lattice spacing

$$\epsilon_a^2 = \frac{1}{4\pi}\frac{a^2}{w_0^2} \qquad \delta_a = a_2\varepsilon_a^2 + b_4\varepsilon_a^2\varepsilon_\pi^2 + a_4\varepsilon_a^4 + [a_1\sqrt{4\pi}\varepsilon_a + s_2\alpha_S\alpha_a^2]$$

Finite volume

$$\delta_L = \frac{8}{3}\varepsilon_\pi^2\,\underline{\left[g_0^3F_1(m_\pi L) + g_0F_3(m_\pi L)\right]} + f_3\varepsilon_\pi^3F_1(m_\pi L)$$

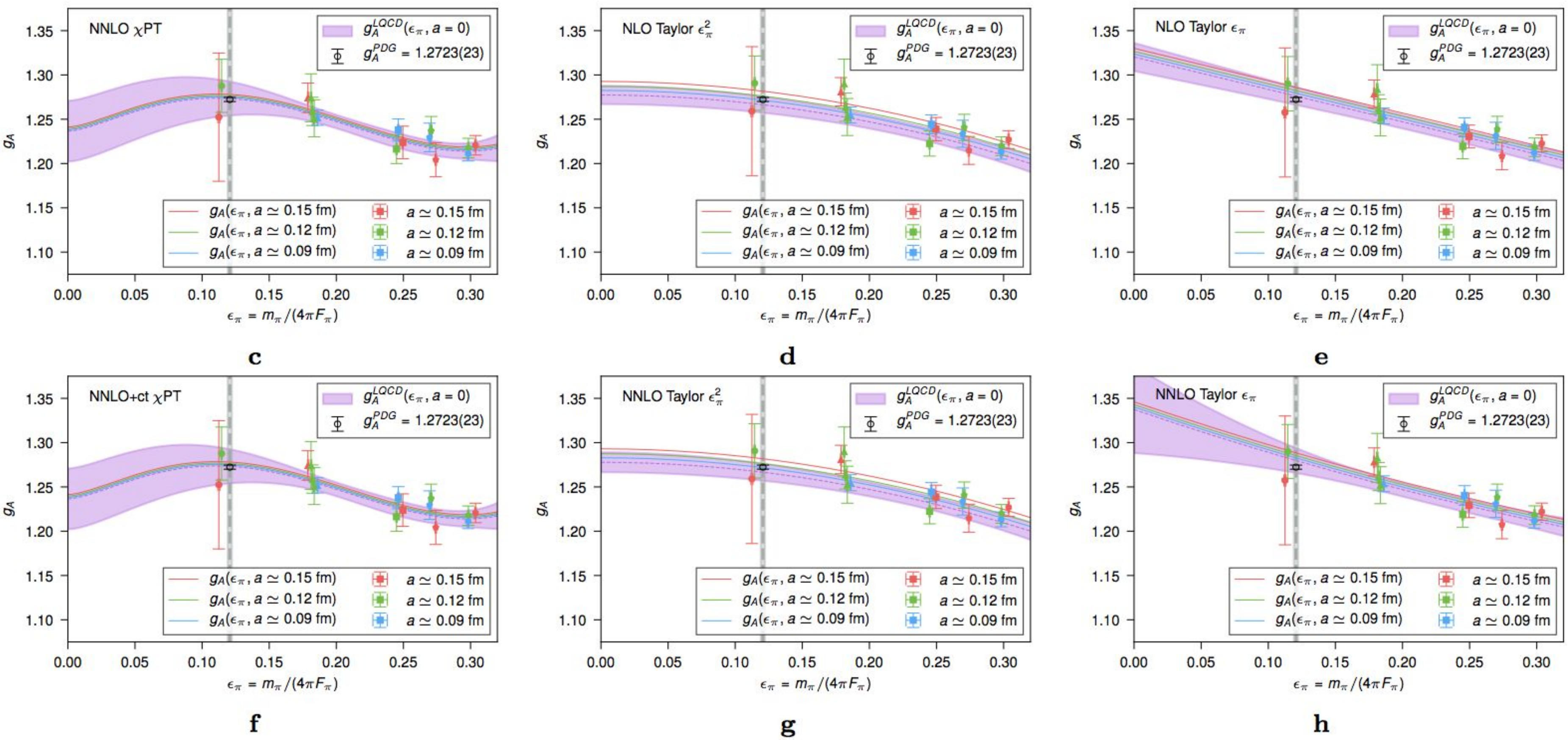
Beane and Savage PRD 70 (2004) 074029

Chiral

$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi} \qquad g_A = g_0 + c_2\epsilon_\pi^2 - \epsilon_\pi^2\left(g_0 + 2g_0^3\right)\ln\left(\epsilon_\pi^2\right) + g_0c_3\epsilon_\pi^3$$

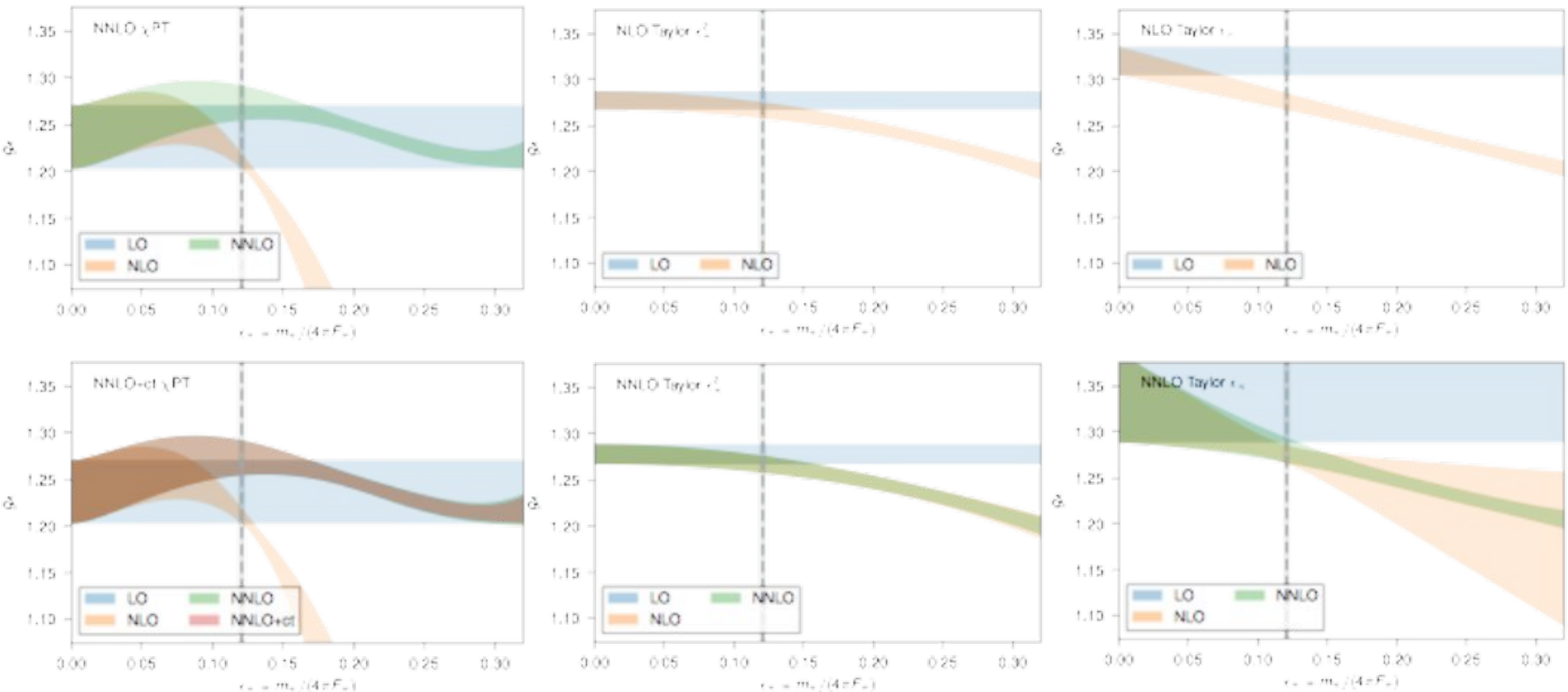
Axial coupling from CalLat and friends

Worked hard at ensuring stability in fits: chiral fits



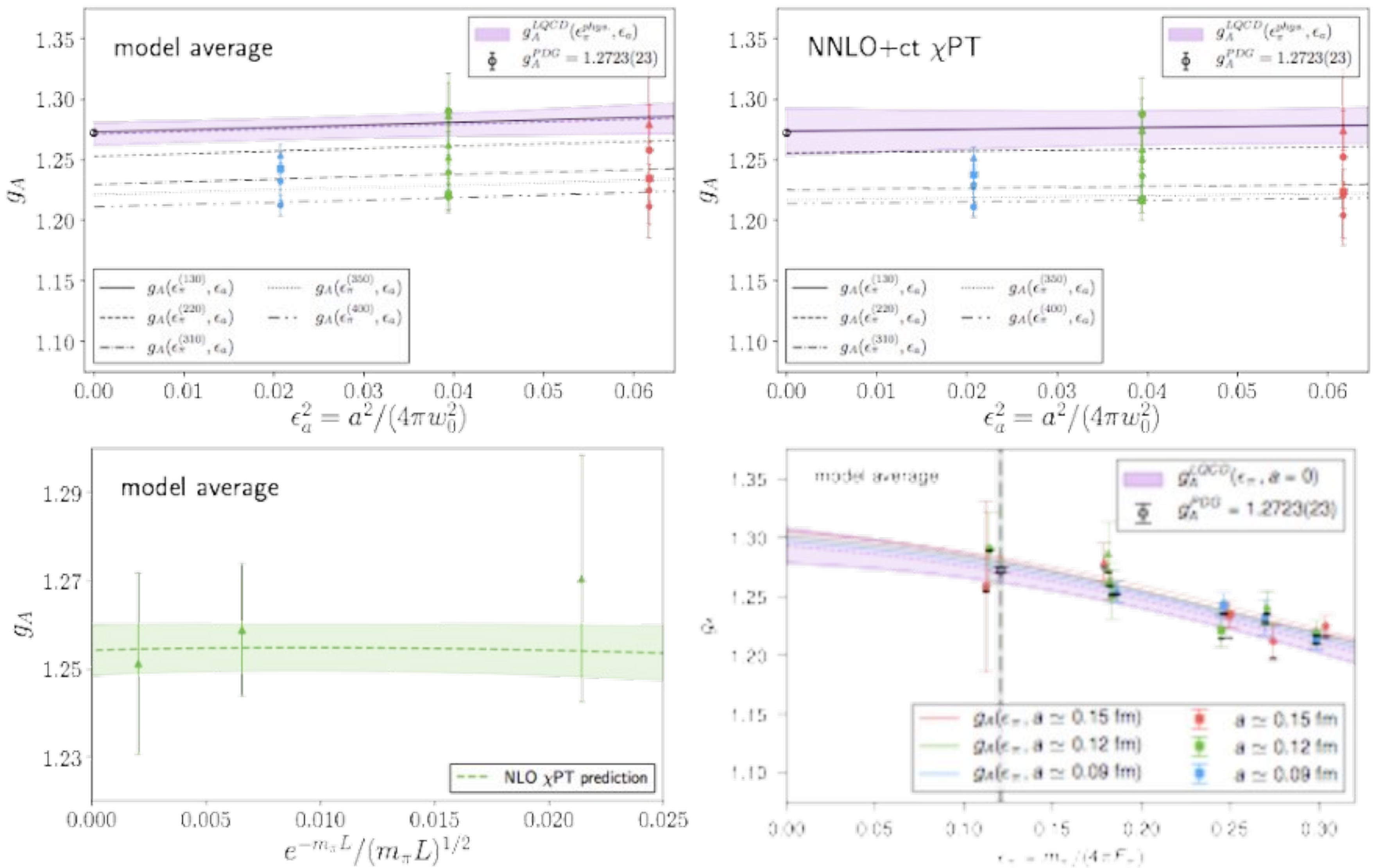
Axial coupling from CalLat and friends

Worked hard at ensuring stability in fits: chiral expansion convergence



Axial coupling from CalLat and friends

Worked hard at ensuring stability in fits: continuum and infinite volume fits



Axial coupling from CalLat and friends

Worked hard at ensuring stability in fits: model average